

# **World-Volume Fields and Background Coupling of Branes**

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*Received September 15, 1998*

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After giving a brief introduction to the concept of branes and their role in string theory, this paper describes a method for formulating the dynamics of branes, especially those containing nonscalar moduli. Emphasis is put on the coupling of branes to fields in the low-energy background supergravity theories, and on preservation of maximal amount of manifest symmetry. The presentation is aimed at physicists who are not experts in string theory.

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## **1. INTRODUCTION**

Extended supersymmetric objects arise as solutions in the low-energy effective field theories of superstring theory/M-theory. Some of them are naturally interpreted as solitons, and carry information about the nonperturbative behavior of the theory, while some bear signs of being elementary excitations (with respect to some perturbative formulation). Most profoundly, the solitons of type II string theory are realized as D-branes, *i.e.*, objects on which elementary strings may end. Many aspects of the dynamics of these objects, which is the main subject of this paper, are by now fairly well understood. Although a fundamental microscopic formulation of the theory is still missing, the evidence is overwhelming that the different perturbative superstring theories, their low-energy supergravity theories, and 11-dimensional supergravity are different manifestations of one underlying nonperturbative theory.

There is limited space here to discuss the roles that the different elementary and solitonic objects have in the various duality transformations between different perturbative theories. We refer to ref. 1. Some will be mentioned

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briefly, but the emphasis is put on the description of the dynamics of branes occurring in superstring theory/M-theory.

## 2. CHARGED BRANES

The way to describe brane dynamics is to formulate actions fulfilling certain requirements. In this section, some things will be said about the basic ingredients of these actions. The methods will later be refined. We let the dimensionality of the brane world-volume be  $p + 1$  and that of space-time  $D$ .

The first invariant considered as (a term in) the action for a brane is the volume. It may be written

$$\int_{\Sigma_{p+1}} d^{p+1} \xi \sqrt{-g} \quad (2.1)$$

where  $g_{ij} = \partial_i X^m \partial_j X^n g_{mn}$  is the induced metric on the world-volume. For dimensionality reasons, it must be multiplied by some dimensional constant, the brane tension. There is also a possibility that some scalar field (if the background theory contains one) will enter inside the integral. This term is referred to as the Nambu–Goto action.

If the spectrum of the background supergravity contains a  $(p+1)$ -form potential  $C_{(p+1)}$  with field strength<sup>2</sup>  $H_{(p+2)} = dC_{(p+1)}$ , a  $p$ -brane can couple electrically to  $C$  via a term in the action<sup>3</sup>

$$q \int_{\Sigma_{(p+1)}} C_{(p+1)} \quad (2.2)$$

where  $q$  is the charge of the brane. As long as the brane has no boundary, this term is clearly invariant under gauge transformations  $\delta_\Lambda C = d\Lambda$ . Together with the kinetic term for  $C$ , we can rewrite the relevant part of the action as an integral over the entire space-time  $\mathcal{M}_D$  as

$$\int_{\mathcal{M}_D} \left( \frac{1}{2} H \wedge *H + q \hat{n} \wedge C \right) \quad (2.3)$$

where  $\hat{n}$  is the  $(D-p-1)$ -form whose components are those of an epsilon tensor normal to the brane and which has  $\delta$ -function support on the world-volume. The equation of motion for  $C$  derived from Eq. (2.3) is modified to include an electric source term (the exact sign here depends on conventions and on

<sup>2</sup>This relation between potential and field strength will later in some cases contain additional terms; in such cases one has to be careful about what exactly is meant by charge.

<sup>3</sup>Of course, not only electric coupling is formulated this way; magnetic coupling is achieved as electric coupling to a dual field strength  $\tilde{C}_{(D-p-3)}$ .

the degrees of the forms) and the brane charge is given as the integral of  $*H$  over any topological  $(D-p-2)$ -sphere surrounding the brane (see Fig. 1),

$$\int_{S_{D-p-2}} *H = \int_{B_{D-p-1}} d*H = \int_{B_{D-p-1}} q \hat{n} = q \quad (2.4)$$

It is important to note that  $q$  is not a “charge density” on the brane, but an object intrinsically defined only for branes of dimension  $p + 1$ .

From what has been said above, it is clear that a charged  $p$ -brane may exist if the background supergravity theory contains an antisymmetric tensor potential of rank  $p + 1$ . It is therefore sensible to look for solutions of the equations of motion for the background supergravity fields that in some sense are localized on a “brane,” *i.e.*, a lower dimensional hypersurface, and where the antisymmetric tensor carries some field strength corresponding to a charge of that hypersurface. It turns out that such solutions exist for all antisymmetric tensors appearing in supergravity theories. For simplicity, we will concentrate on maximally supersymmetric theories in a maximum number of uncompactified dimensions, which leaves us with type IIB supergravity in 10 dimensions and 11-dimensional supergravity (type IIA in 10 dimensions) may be obtained

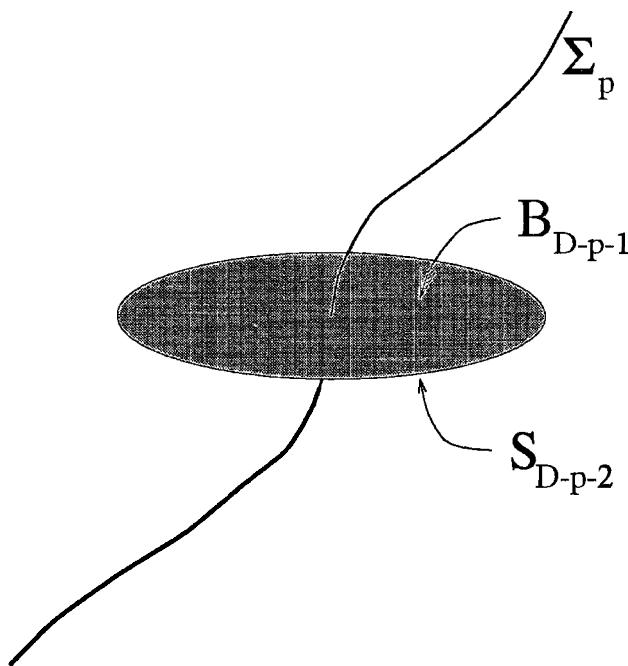
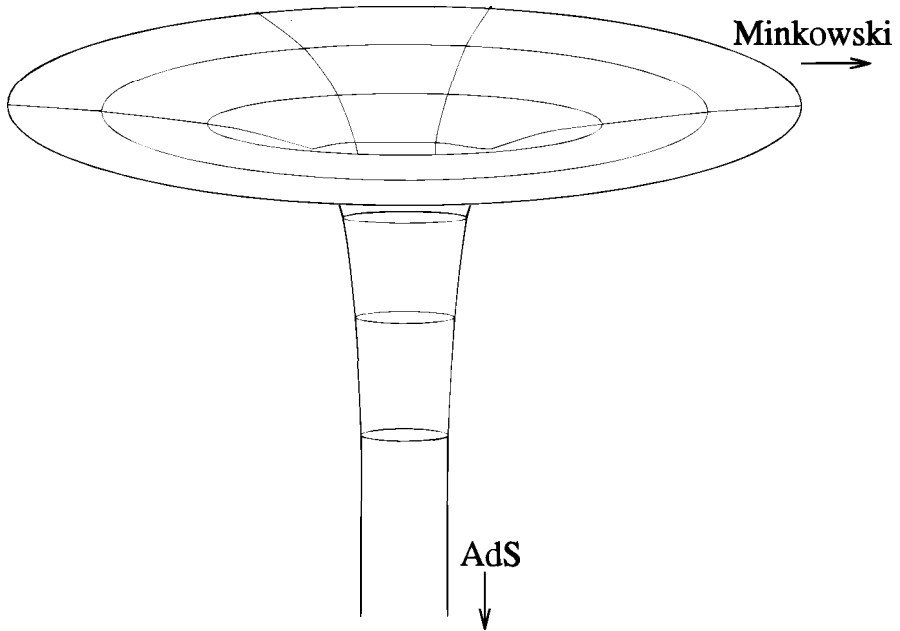


Fig. 1. The topology in connection with brane charges. The time direction is not in the figure.

as a dimensional reduction from 11 dimensions, and we do not consider it separately here).<sup>4</sup> Type IIB supergravity is the low-energy effective theory for type IIB superstring theory, while 11-dimensional supergravity is related to the so-called M-theory, whose fundamental degrees of freedom may reside in the 11-dimensional supermembrane.

As an example of how brane solutions behave, we can take a look at the tensorial branes of  $D = 11$  supergravity, a membrane and a 5-brane (see following section). The “electric” membrane solution contains a singularity, which is typical for an elementary excitation (such as an ordinary electrically charged particle)—it acts as a source. The “magnetic” 5-brane solution, on the other hand, is nonsingular and in this sense exhibits a typically solitonic behavior. Even though solitonic solutions are nonsingular, the charge is protected by the topology of space-time.

Around brane solutions in supergravity (Fig. 2), there will be zero modes, roughly speaking corresponding to flat directions in a moduli space of such



**Fig. 2.** A rough picture of the spacelike geometry associated with a brane solution in supergravity.

<sup>4</sup>It should be noted that in addition to branes that carry tensorial charges, there are configurations that are gravitationally charged (Kaluza–Klein monopoles, gravitational waves [2]). Although the discussion here is limited to tensorial branes, compactification/dimensional reduction generically mixes the two categories, since components of the metric appear as tensors in the lower dimensionality.

solutions. There has been surprisingly little work done on such zero modes and the derivation of brane dynamics from them [3], as compared to the situation in field theory, where such methods are widely used for, *e.g.*, monopoles.

An important property of the relevant branes is that they are stable. Exactly as for solitons in supersymmetric field theory (monopoles . . .), this is ensured by the BPS (Bogomol’nyi–Prasad–Sommerfeld) property, which in the simplest case implies a relation between mass and charge,  $m \sim q$ . In supersymmetric theories, such a relation is exactly what is needed for the existence of “short multiplets,” smaller representations of the supersymmetry algebra than the generic ones. The argument is roughly that since the number of states in a multiplet should not get renormalized, the BPS property is exact. BPS states are generated by a fraction of the supersymmetry generators, which for single branes (the only case treated here) is 1/2, but for multiple-brane configurations may be a smaller fraction.

The main aim of this paper is to describe the intrinsic dynamics of the branes. They will be described by massless fields on the brane world-volumes, belonging to supermultiplets generated by half the number of space-time supersymmetry generators, due to the BPS property.

### 3. SUPERGRAVITIES AND BRANES

We first list the field content of the relevant supergravity theories. In type II 10-dimensional supergravities there are 128 bosonic and 128 fermionic physical degrees of freedom [the numbers below refer to the dimensions of the representations of the transverse rotation group  $\text{Spin}(D - 2)$ ] forming short ( $2^8$ ) supermultiplets:

$g$	35	}	NS-NS
$B_{(2)}/\tilde{B}_{(6)}$	28		
$\phi$	1		
<b>IIA</b>			
$C_{(1)}/\tilde{C}_{(7)}$	8	}	RR
$C_{(3)}/\tilde{C}_{(5)}$	56		
<b>IIB</b>			
$C_{(0)}/\tilde{C}_{(8)}$	1	}	NS-R, R-NS
$C_{(2)}/\tilde{C}_{(6)}$	28		
$C_{(4)}(\text{chiral})$	35		
$\Psi_{\mu}^{\alpha}$	56	}	NS-R, R-NS
$\Psi_{\mu\alpha}$	56		
$\lambda_{\alpha}$	8		
$\lambda^{\alpha}$	8		
		$\Psi_{i\mu}^{\alpha}$	$2 \times 56$
		$\lambda_{i\alpha}$	$2 \times 8$

The fields in  $D = 11$  supergravity are

$g$	44
$C_{(3)}/\tilde{C}_{(6)}$	84
$\Psi_{\mu}^{\alpha}$	128

Here,  $g$  denotes the metric, the  $B$ 's and  $C$ 's are antisymmetric tensor potentials, and their dual potentials are denoted with a tilde. The rank of the tensors is given in parentheses. The  $\Psi$ 's are spin-3/2 gravitino fields and the  $\lambda$ 's spinors. NS and R denote the origin of the fields in perturbative string theory, where they denote spin structures for left- and right-moving fields on the string world-sheet.

From the table it is straightforward to read off the dimensionalities of the different possible branes<sup>5</sup>:

$$\mathbf{IIB}: \quad p = (-1), 1 \text{ (2 charges)}, 3, 5 \text{ (2 charges)}, 7, (9)$$

$$\mathbf{IIA}: \quad p = 0, 1, 2, 4, 5, 6, (8)$$

$$\mathbf{D} = \mathbf{11}: \quad p = 2, 5$$

We know that the degrees of freedom of the branes will form BPS multiplets on the brane world-volumes. There are several ways of deducing exactly what fields these multiplets consist of. Here we will make a very naive counting argument. First, count the number of fermionic degrees of freedom. The background theory has (in all cases) 32 supercharges. Out of these, only 16 generate physical states. On the brane, this BPS property manifests itself as a fermionic gauge symmetry, “ $\kappa$ -symmetry,” gauging away half the spinor. Then, accounting for the equation of motion for the world-volume fermions, this number is further reduced to 8. Therefore, there must also be 8 bosonic degrees of freedom on the world-volume. Some of these are obvious. There are translational degrees of freedom, corresponding to moving the brane in a transverse direction. They make up  $D - p - 1$ , and there remain

$$8 - (D - p - 1) = \begin{cases} p - 1 & (D = 10) \\ p - 2 & (D = 11) \end{cases} \quad (3.1)$$

These numbers match with a world-volume  $U(1)$  vector potential ( $p - 1$  degrees of freedom in  $p + 1$  dimensions) in  $D = 10$  and a chiral 2-form (3 degrees of freedom) for the 5-brane in  $D = 11$ . It should perhaps be mentioned that this crude approach misses out some interesting aspects, particularly in relation to the reduction from 11 to 10 dimensions, but we try to keep it on a simple level.

<sup>5</sup>With tensorial charges; the gravitational ones are again left out, resulting in an apparent mismatch between type IIA and the reduction of  $D = 11$ .

#### 4. D-BRANE ACTIONS

We now turn to the issue of formulating brane dynamics via an action principle. In the discussion above concerning the existence of tensorially charged branes, no distinction was made between different “types” of branes. From the point of view of a perturbative string theory, there is a clear difference between fields in the NS-NS sector and in the RR sector. A fundamental (perturbative) superstring couples only to NS-NS fields. It was unclear for a long time how coupling to RR fields was to be achieved. The solution, due to Polchinski [4], is that the supergravity solutions corresponding to RR-charged branes are nonperturbative “states” (using a somewhat sloppy language—the solutions are of course classical, and one should consider states in the corresponding quantum field theory) in the superstring theory. As argued in the previous section, they have massless excitations corresponding to a world-volume vector. This vector couples to endpoints of open strings. The open strings are thus forced to end on the brane, *i.e.*, to have Dirichlet boundary conditions.

Let us first consider actions for these D-branes, and later turn to improvements of this description. Some essential ingredients are present in Section 1. One piece in the action should be the invariant volume, as in Eq. (2.1), and another one the tensor coupling of Eq. (2.2) (for simplicity, tension factors are suppressed). The entire action should be some modification of the combination of these terms, also containing the world-volume vector and maybe a dilaton coupling.

Some information about the way the vector enters may be obtained from gauge invariance in the configuration depicted in Fig. 3. If we use the knowledge that the fundamental superstring couples to the NS-NS 2-form potential  $B$  with field strength  $H = dB$ , we can calculate the variation of its action under a gauge transformation  $\delta B = d\Lambda$ :

$$\delta \int_{\Sigma_2} B = \int_{\Sigma_2} d\Lambda = \int_{\partial\Sigma_2} \Lambda \quad (4.1)$$

( $\partial\Sigma_2$  is the dotted line in Fig. 3). This variation is canceled if the string endpoint is considered as a point source on the world-volume, so that there is a term  $\int_{\partial\Sigma_2} A$ , provided that the world-volume vector potential  $A$  transforms as  $\delta A = \Lambda$  under these gauge transformations in the background theory. The gauge-invariant [both background and world-volume U(1)] field strength of  $A$  must then be  $F = dA - B$ , and this is the only combination through which  $A$  can enter the brane action.

As is known from type IIA/B supergravity, the RR tensor fields fulfill Bianchi identities which are “modified” [5]. If all the RR forms (odd for

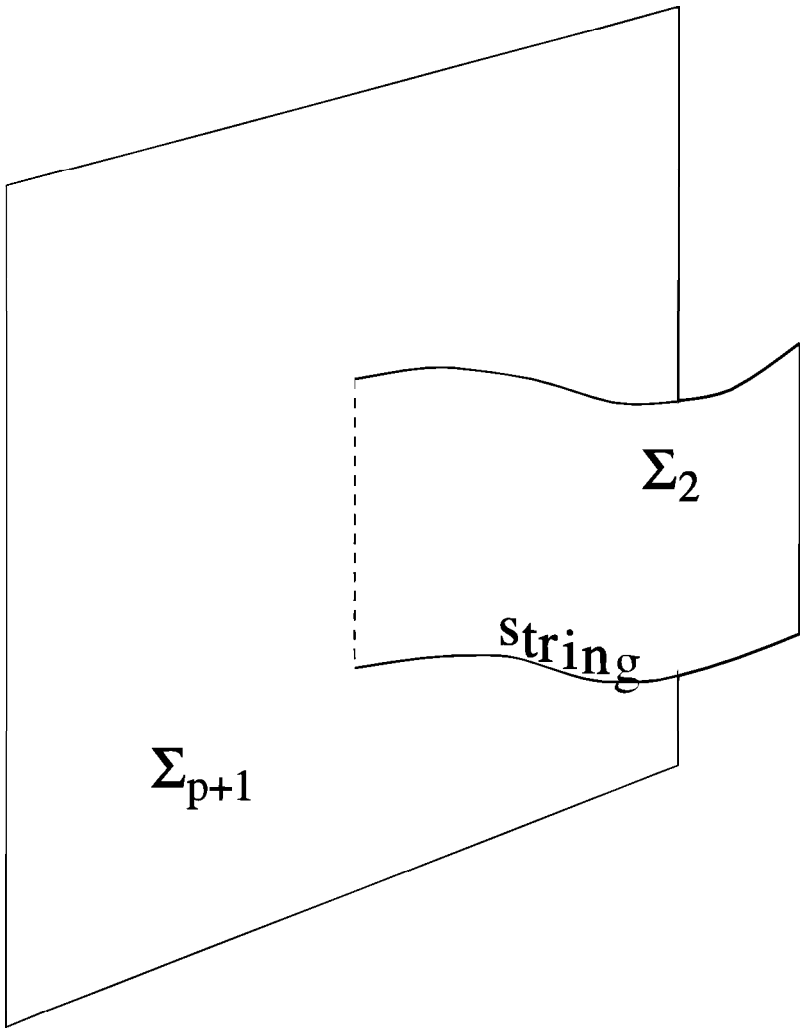


Fig. 3. A string attached to a D-brane.

type IIA, even for type IIB) are collected in  $C = C_{(r)} \oplus C_{(r+2)} \oplus C_{(r+4)} \oplus \dots$ , where  $r = 1$  for IIA and 0 for IIB, the field strengths are

$$R = e^B d(e^{-B} C) = dC - H \wedge C \quad (4.2)$$

It is straightforward to verify that gauge invariance demands the Wess–Zumino term to be modified as



$$\int_{\Sigma_{p+1}} e^F C \quad (4.3)$$

A  $\beta$ -function calculation [6] for the string ending on the D-brane (one  $\beta$ -functional for each NS-NS background field, which act as “coupling constants”) shows that the  $\beta$ -functions may be canceled by the classical variation of

$$- \int_{\Sigma_{p+1}} d^{p+1} \xi e^{-\phi} \sqrt{-\det(g + F)} \quad (4.4)$$

which is therefore considered as the appropriate modification of the volume (“kinetic”) term in the D-brane action. This term is known as the Dirac–Born–Infeld (DBI) action.

We have now, through different arguments, arrived at the final form [7] of the action for the bosonic degrees of freedom of a D-brane:

$$S = - \int_{\Sigma_{p+1}} d^{p+1} \xi e^{-\phi} \sqrt{-\det(g + F)} + \int_{\Sigma_{p+1}} e^F C \quad (4.5)$$

How is this made supersymmetric? The answer is straightforward: the bosonic world-volume is embedded in superspace instead of in ordinary space. All background fields become superfields, and enter the action (4.5), which is still formally valid, as pullbacks from superspace to the world-volume. Thus also the fermionic coordinates in the background superspace become dynamical variables. Their number is 32, and as mentioned earlier, there must be a fermionic gauge symmetry to take away half of them, in accordance with the BPS property of the brane. The symmetry is known as  $\kappa$ -symmetry, and its construction and verification is a most central part of the formulation of the dynamics of a super-D-brane. Let the superspace coordinates be  $Z^M = (x^m, \theta^\mu)$ , and the locally inertial indices be  $A = (a, \alpha)$ . The  $\kappa$ -symmetry is most covariantly formulated as a local fermionic translation  $\delta_\kappa Z^M = \kappa^\alpha E_\alpha^M$ , where  $E_A^M$  is the super-vielbein. If this symmetry is to remove only half a spinor, the spinor  $\kappa$  must be subject to the relation  $\kappa = P\kappa$ , where  $P$  is a projection operator of half the maximal rank. We will not go through the details of the construction, but refer to ref. 8. The projection operator may be written as  $P = \frac{1}{2}(1 + \Gamma)$ , where  $\Gamma^2 = 1$ , and takes the schematic form (apart from numerical and dilaton factors)

$$\Gamma d^{p+1} \xi \sim \frac{1}{\sqrt{-\det(g + F)}} (\gamma_{(p-1)} + F \wedge \gamma_{(p+1)} + F \wedge F \wedge \gamma_{(p-3)} + \dots) \quad (4.6)$$

the  $\gamma_{(n)}$ s being  $n$ -forms whose components are antisymmetric products of  $\gamma$ -matrices.

## 5. MORE COVARIANCE IN TYPE IIB

In spite of the success of the program presented in the previous section, some questions remain. Some concern mainly type IIB, and some tensorial branes in general.

- Type IIB supergravity has an  $SL(2; \mathbb{R})$  invariance. The subgroup  $SL(2; \mathbb{Z})$  is an S-duality (strong–weak coupling symmetry) of type IIB superstring theory. It is obscured by the (perturbative) division into NS-NS and RR sectors. Can it be manifested?
- What is special about the  $B$  field? Could not any  $n$ -form potential  $C$  enter brane actions through an  $n$ -form world-volume field strength  $F \sim dA - C$ ?

The answer to the second question is in the affirmative. As we will see, it also contains the answer to the first question.

The asymmetry pointed out here can be illustrated by the actions for a fundamental string and a D-string in type IIB. They should be related by transformations in  $SL(2; \mathbb{Z})$ . The fundamental superstring is described by (from now on, all world-volumes are considered to be embedded in superspace)

$$S = - \int_{\Sigma_2} d^2\xi \sqrt{-g} + \int_{\Sigma_2} B \quad (5.1)$$

while the D-string action is obtained from Eq. (4.5) as

$$S = - \int_{\Sigma_2} d^2\xi e^{-\phi} \sqrt{-\det(g + F)} + \int_{\Sigma_2} (C_{(2)} + FC_{(0)}) \quad (5.2)$$

The vector potential on the D-brane world-volume carries no local degrees of freedom. There are, however, global ones, namely a quantized electric flux [9], giving charge with respect to  $B$  (this is most easily seen in a canonical treatment). So while the fundamental string has charges  $\vec{q} = (1, 0)$  with respect to  $(B, C)$ , the D-brane action describes all the sectors with  $\vec{q} = (m, 1)$  for integer  $m$ 's.

The asymmetry is cured by the introduction of yet another world-volume vector potential. The two vectors now come in a doublet of  $SL(2; \mathbb{R})$ , exactly as in the background supergravity. One has to rescale the metric, since the one occurring so far was the “string metric,” whose curvature scalar is multiplied by a dilaton factor in the supergravity action. The  $SL(2)$ -invariant

metric is the ‘‘Einstein metric,’’ *i.e.*, the one that comes without dilaton factors in the Einstein–Hilbert action. They are related by

$$g_{\text{string}} = e^{\phi/2} g_{\text{Einstein}}$$

The scalar fields  $\phi$  and  $C_{(0)}$  parametrize the coset  $SL(2; \mathbb{R})/U(1)$ , and their covariance must also be taken care of. The world-volume field strengths are most conveniently combined into a complex 2-form  $F$ , which is invariant under  $SL(2)$  and also contains the scalars. We refer to ref. 10 for details. The action turns out to be astonishingly simple:

$$S = - \int_{\Sigma_2} d^2\xi \lambda (1 - *F*\bar{F}) \quad (5.3)$$

We note that there is no Wess–Zumino term in this action, which is logical since the tensors should not occur twice. The field  $\lambda$  is a Lagrange multiplier. It is straightforward to verify that Eq. (5.3) gives the correct coupling to the background fields and yields the correct equations of motion, *i.e.*, equivalent to the string actions above in those charge sectors, and that it enjoys  $\kappa$ -symmetry. Due to flux quantization, the action describes the entire set of strings with integer charges  $\vec{q} = (m, n)$  [this is actually more than the orbit of  $\vec{q} = (1, 0)$  under  $SL(2; \mathbb{Z})$  consisting only of coprime pairs]. For constant (or slowly varying) scalar fields, one also verifies the correct string frame tension [11]

$$T = \sqrt{e^{-2\phi} n^2 + (m + nC_{(0)})^2} \quad (5.4)$$

It is conceivable that the action (5.3) can be taken as a starting point for the formulation of an  $SL(2; \mathbb{Z})$ -covariant type IIB superstring perturbation theory, where the quantized fluxes are conserved when strings split or join. The situation seems much more hopeful than for the S-duality of  $N = 4$  super-Yang–Mills theory, where duality mixes electric and magnetic charges. This is not the case for type IIB, where charges with respect to two different fields are mixed (the analogy of a magnetically charged object is a 5-brane).

## 6. HIGHER DIMENSIONAL BRANES

How can the picture of last section be generalized to other branes? The schematic ansatz is that all coupling to background tensors should be achieved through world-volume field strengths  $F \sim dA - C$ . The properties of the world-volume fields should mimic the background ones, so that for type IIB one would have a complex  $F_{(2)}$ , a real  $F_{(4)}$ , a complex  $F_{(6)}$ , and so on.

One problem presents itself immediately: while the vector potential on the string world-sheet did not carry any local degrees of freedom, these

potentials in general will, and one naively gets to many physical degrees of freedom. The solution is that there will be an algebraic relation, a generalized self-duality, between the field strengths. We will illustrate this first in the case of the type IIB 3-brane.

The type IIB 3-brane is known to be “self-dual” [12], in the sense that a duality transformation of  $F$  (which is necessarily nonlinear, since the conjugate  $E$ -field obtained from the DBI action is nonlinear) accompanied by a  $\mathbb{Z}_2$  in  $SL(2; \mathbb{Z})$  leaves the action invariant. What one now has to do is to find the appropriate construction of the world-volume field strengths  $F_{(2)}$  (complex) and  $F_{(4)}$  (real). This is achieved through consideration of gauge invariance. It is amusing to note that the modified Bianchi identities of the background supergravity force the world-volume Bianchi identities to behave similarly (all details are found in ref. 13). With some guidance from the string case, a reasonable ansatz for an action is

$$S = \int_{\Sigma_4} d^4\xi \lambda(1 + \Phi(F_{(2)}, \bar{F}_{(2)}) - (*F_{(4)})^2) \quad (6.1)$$

with  $\Phi$  being some yet unknown function. There are a number of requirements which all turn out to point to the same answer (in terms of  $\Phi$ ). One is that the exact form of the self-duality relation imposed on  $F_{(2)}$  is to be consistent with the coupling to the background fields; it is not *a priori* clear that equations of motion and Bianchi identities contain identical background tensors. Another is that the action should be  $\kappa$ -symmetric. Using this criterion leads to the conclusion that  $\kappa$ -symmetry demands a specific form of the self-duality, which is identical to the one obtained from the first requirement. Very schematically, the form of the self-duality reads

$$i*F_{(4)}*F_{(2)} \sim F_{(2)} + F_{(2)}^2\bar{F}_{(2)} \quad (6.2)$$

corresponding to a  $\Phi$  with quadratic and quartic terms. To linear level, Eq. (6.2) reduces to the ordinary complex self-duality, implying that one real component of the complex  $F$  is the dual of the other. At the price of breaking the  $SL(2)$ -covariance, one real vector potential may be eliminated, and one is back at the field content of the DBI action (if also the nondynamical  $F_{(4)}$  is solved for).

Partial results have been obtained for the multiplet of  $(m, n)$ -type IIB 5-branes, but the work is not finished. It involves an interesting duality relation between a complex 2-form and a real 4-form, some aspects of which were described in ref. 13. There is a possibility that the problem is algebraically intractable [15]. The 5-brane in 11-dimensional supergravity has been described in an analogous way [14].

A drawback of these formulations is of course that the self-duality does not follow from the action principle, but has to be supplemented by hand. For the 11-dimensional 5-brane, there is no action, since the multiplet contains a chiral 2-form; for the branes in type IIB it is the price one has to pay in order to keep the S-duality symmetry. An advantage is that the actions, as well as the self-duality relations, become polynomial, instead of taking the nonpolynomial DBI form. A great advantage of the formulation is that it provides a natural framework for studying branes, other than strings, ending on branes and coupling via their boundaries to the world-volume tensor potentials, analogously to the way a string ends on a D-brane (it may be seen as surprising that the formulation exists for the 5-brane in  $D = 11$ , since 5-branes do not end on 5-branes). This aspect has not been worked out in detail, but looks promising.

## ACKNOWLEDGMENTS

Different parts of the work mentioned here were done in collaboration with Tom Adawi, Ulf Gran, Alexander von Gussich, Magnus Holm, Bengt E. W. Nilsson, Behrooz Razaznejad, Per Sundell, Paul K. Townsend, and Anders Westerberg. I would also like to thank the organizers of the workshop Quantum Gravity in the Southern Cone in San Carlos de Bariloche for their hospitality and helpfulness during my stay in Argentina.

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